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UNIVERSITY COLLEGE
TATI

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET	
COURSE CODE	: BGE 1123
COURSE	: ALGEBRA
SEMESTER/SESSION	: 1-2023/2024
DURATION	: 3 HOURS

Instructions:

1. This booklet contains **6** questions in SECTION A, **3** questions in SECTION B and **2** questions in SECTION C. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 6 PRINTED PAGES INCLUDING COVER PAGE

SECTION A (50 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Given matrix $A=\begin{pmatrix} 2 & q & 3 \\ 5 & -1 & 7 \\ r-3 & r+s & 1 \end{pmatrix}$ and $B=\begin{pmatrix} 2 & 1 & 3 \\ 5 & -1 & 7 \\ -1 & 1 & 1 \end{pmatrix}$.

a) If $A=B$, find the value of q, r and s. (3 marks)

b) Based on the value, find A^{-1} by using adjoint method. (6 marks)

QUESTION 2

Given $(x-2)$ is a factor of polynomial $f(x)=3x^3-10x^2+kx-2$.

a) Find the value of k. (3 marks)

b) Hence, find the remainder when $f(x)$ is divided by $(2x+1)$. (3 marks)

QUESTION 3

Solve θ for the following trigonometric equation in the interval of $0° \le \theta \le 360°$.

a) $\sin\theta = -\frac{1}{2}$ (2 marks)

b) $4\tan\theta - 8 = -4$ (3 marks)

c) $3\cos 2\theta + 4 = 2$ (6 marks)

QUESTION 4

Given $f(x)=2-x^2$ and $g(x)=\sqrt{2x-4}$.

a) Determine the domain for $g(x)$. (1 mark)

b) Find $(f\circ g)(x)$. (2 marks)

c) Find $(g\circ g)(5)$. (2 marks)

QUESTION 5

Given $z_1 = 2+i$ and $z_2 = 3-4i$, find $\frac{1}{z_1}+\frac{1}{z_2}$ in the form of $a+bi$. (5 marks)

QUESTION 6

Given three vectors, $\vec{u} = \mathbf{i}+3\mathbf{j}$, $\vec{v} = 2\mathbf{i}+2\mathbf{j}+5\mathbf{k}$ and $\vec{w} = \mathbf{i}-4\mathbf{j}-2\mathbf{k}$. Find each of the following:

a) $\vec{v}\cdot\vec{w}$. (2 marks)

b) the angle between $\vec{v}$ and $\vec{w}$. (3 marks)

c) $\vec{u}\cdot(3\vec{u}-2\vec{v})$. (3 marks)

d) the area of parallelogram formed by $(\vec{u}+\vec{v})$ and $\vec{w}$. (6 marks)

SECTION B (30 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

a) Given a triangle with $a=12$, $b=10$ and $c=6$. Solve the triangle by finding the value of all angles. (6 marks)

b) Prove that $(1-\sin\theta)(1+\csc\theta)=\cos x\cot x$. (5 marks)

QUESTION 2

a) Use De Moivre's theorem to simplify $\left(\sqrt{3}-i\right)^{12}$. (5 marks)

b) Find the value of x and y if $(3+4i)^2-2(x-iy)=x+iy$. (6 marks)

QUESTION 3

Express the following improper fraction in terms of partial fractions.

$$\frac{x^3+x^2+3x+7}{x^2-1}$$

(8 marks)

SECTION C (20 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

In triangle PQR, the measure of angle Q is three times the measure of angle P. The measure of angle R is $20°$ more than the measure of angle P. Write down a system of linear equations based on the information given. Then, find the measure of each angle by using the Gauss Elimination method. (10 marks)

QUESTION 2

Given the functions $g(x)=\frac{1}{2x-5}$.

a) Find the domain and range of $g(x)$. (3 marks)

b) Sketch the graph of $g(x)$. (2 marks)

c) Show that $g(x)$ is one-to-one function and explain your answer. Hence, find $g^{-1}(x)$. (5 marks)

---**END OF QUESTION**----------------------------------

FORMULA

$$A^{-1}=\frac{1}{|A|}adj(A)$$

$$adj(A)=\left[A_{ij}\right]^T$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}=ad-bc$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$H^2=A^2+O^2$$

$$(a-b)(a+b)=a^2-b^2$$

$$\tan\theta=\frac{\sin\theta}{\cos\theta}$$

$$\cot\theta=\frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta+\cos^2\theta=1$$

$$1+\tan^2\theta=\sec^2\theta$$

$$\csc\theta=\frac{1}{\sin\theta}$$

$$\cos\theta=\frac{1}{\sec\theta}$$

$$1+\cot^2\theta=\csc^2\theta$$

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$Area=\frac{1}{2}ab\sin C$$

$$r=|z|=\sqrt{\left(a^2+b^2\right)}$$

$$\theta=Arg(z)=\tan^{-1}\left|\frac{b}{a}\right|$$

$$z=r(\cos\theta+i\sin\theta)$$

$$z^n=r^n(\cos n\theta+i\sin n\theta)$$

$$z=re^{i\theta}$$

$$i^2=-1$$

$$|\vec{u}|=\sqrt{a^2+b^2+c^2}$$

$$\hat{u}=\frac{\vec{u}}{|\vec{u}|}=\frac{ai+bj+ck}{\sqrt{a^2+b^2+c^2}}$$

$$\vec{u}\cdot\vec{v}=a_1a_2+b_1b_2+c_1c_2$$

$$\theta=\cos^{-1}\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\right)$$

$$\vec{u}\times\vec{v}=\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}i-\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}j+\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}k$$

$$=(b_1c_2-b_2c_1)i-(a_1c_2-a_2c_1)j+(a_1b_2-a_2b_1)k$$

$$Area=|\vec{u}\times\vec{v}|$$